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## Conformal Non-Geometric Gravity in Six Dimensions and M-Theory above the Planck Energy

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### ABSTRACT

The proposal that a strong coupling limit of the five-dimensional type II string theory (M-theory compactified on a 6-torus) in which the Planck length becomes infinite could give a six-dimensional superconformal phase of M-theory is reviewed. This limit exists for the free theory, giving a 6-dimensional theory with (4,0) supersymmetry compactified on a circle whose radius gives the 5-dimensional Planck length. The free 6-dimensional theory has a fourth rank tensor gauge field with the symmetries of the Riemann tensor instead of a symmetric tensor gauge field, but its dimensional reduction gives conventional linearised gravity in five dimensions. The possibility of an interacting form of this theory existing and the consequences it would have for the geometric picture of gravity are discussed.

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Gravitational theories are characterised by a length scale  $l$  which determines the strength of gravitational coupling and also the scale at which quantum gravity effects are expected to become important; in M-theory this is the  $D = 11$  Planck length. In recent years there has been great success in describing the strong coupling behaviour of a variety of gauge and string theories in terms of dual theories. Here, a recent proposal [1] (reviewed in [2]) to extend this to gravitational theories will be described. The idea is to consider a suitable limit in which  $l \rightarrow \infty$  so that masses become zero and a phase of the theory with a vast amount of unbroken gauge symmetry is obtained. The specific model is M-theory compactified on  $T^6$  to  $D = 5$ , with a low-energy effective description in terms of  $D = 5, N = 8$  supergravity. The proposal is that the limit  $l \rightarrow \infty$  is a kind of decompactification limit to a six-dimensional theory. This theory is conformally invariant, with no length scales, and the scale  $l$  arises as the compactification scale: compactification from  $D = 6$  on a circle of radius  $R$  gives a theory with  $D = 5$  Planck scale  $l = R$ . If such a superconformal phase of M-theory exists, then it would have many consequences for our understanding of M-theory and gravitational physics. The treatment here will follow [1], finishing with some speculations which will be discussed more fully elsewhere.

The aim is then to find a gravitational analogue of some of the weak-strong coupling dualities that have been found in field theories and string theories. At first sight, it would seem that such dual theories of gravity would be unlikely to exist, but the close relations between gravity and gauge theories and the implications such a dual description could have for quantum gravity suggest that it could be worthwhile to investigate this possibility.

A useful example to try and generalise is 5-dimensional maximally supersymmetric Yang-Mills theory. It is non-renormalisable and so new physics should emerge at short distances. It will be supposed that the  $D = 5$  super-Yang-Mills multiplet arises as part of a consistent theory that can be extrapolated to high energies; for example, it could arise as part of the 5-dimensional heterotic string, or on a stack of D4-branes in the IIA string, and in both these cases there would be in

addition supergravity fields and an infinite number of massive fields in the full theory. The theory has BPS 0-branes given by lifting self-dual Yang-Mills instantons in 4 Euclidean dimensions to soliton world-lines in 4+1 dimensions. These have mass  $M \propto \frac{|n|}{g_{YM}^2}$  where  $n$  is the instanton number, so that these become light as the dimensionful Yang-Mills coupling  $g_{YM}$  becomes large. In [3] it was proposed that these should be interpreted as Kaluza-Klein modes for a  $D = 6$  theory compactified on a circle of radius  $R = g_{YM}^2$ . In the strong coupling limit an extra dimension opens up to give a 6-dimensional (2,0) supersymmetric theory in which the gauge field is replaced by a 2-form gauge field  $B_{MN}$  with self-dual field strength, and the 5 scalar fields are all promoted to scalar fields in 6 dimensions [3-5]. The  $D = 6$  theory is believed to be a non-trivial superconformally invariant quantum theory [5] and the  $D = 5$  gauge coupling arises as the radius of the compactification circle,  $g_{YM}^2 = R$ . The relationship between the  $D = 5$  and  $D = 6$  theories is straightforward to establish for the free case in which the Yang-Mills gauge group is abelian, but in the interacting theory the 6-dimensional origin of the  $D = 5$  non-abelian interactions is mysterious; there are certainly no local covariant interactions that can be written down that give Yang-Mills interactions when dimensionally reduced [6]. Nonetheless, the fact that these  $D = 5$  and  $D = 6$  theories arise as the world volume theories of D4 and M5 branes respectively [7] gives strong support for the existence of such a 6-dimensional origin for the gauge interactions.

The W-bosons and magnetic strings in  $D = 5$  arise from self-dual strings in  $D = 6$ . At the origin of moduli space the W-bosons become massless and the tensions of the self-dual strings in  $D = 6$  must also become zero. The nature of the theory at such points is unclear. For example, one guess might be that it could be described by some kind of string field theory, but it was argued in [5] that such a description would over-count the degrees of freedom. Nonetheless, given that mysterious interactions with no conventional field theory formulation arise in the M5-brane world-volume theory, it is natural to ask whether similar unconventional interactions could arise elsewhere; given that they arise in one corner of M-theory, it seems reasonable to expect that there are other corners in which such phenomena

occur.

The theory that will be considered here is five-dimensional  $N = 8$  supergravity (ungauged), which has a global  $E_6$  symmetry and a local  $Sp(4) = USp(8)$  symmetry [8]. It is non-renormalisable, and will be regarded as arising as a massless sector of some consistent theory, such as M-theory compactified on a 6-torus, in which the global  $E_6$  symmetry is broken to a discrete U-duality subgroup [9]. The massless bosonic fields consist of a graviton, 27 abelian vector fields and 42 scalars. The action is

$$S = \int d^5x \sqrt{-g} \left( \frac{1}{l^3} R - \frac{1}{4l} F^2 + \dots \right) \quad (1)$$

where  $l = \kappa^{2/3}$  is the 5-dimensional Planck length. If this does have a dual at strong gravitational coupling, i.e. a limit as  $l \rightarrow \infty$ , then it is natural to look first for a theory with 32 supersymmetries and with  $E_6 \times Sp(4)$  symmetry, as the simplest possibility would be if these symmetries survived at strong coupling. The arguments used in [1] are similar to those used in [9,10,11] for extrapolating to large values of dimensionless string couplings and moduli, and it will be assumed that all symmetries are preserved and BPS states are protected and survive as the coupling  $l$  is increased. The symmetries then impose stringent constraints on what can happen, and one can then seek checks on the predicted strong-coupling dual.

Comparison with the  $D = 5$  gauge theory case suggests seeking a 6-dimensional theory compactified on a circle in which the 27  $D = 5$  vector fields are replaced with 27 self-dual 2-form gauge fields in  $D = 6$ . Indeed, the decomposition of  $N = 8$  supergravity into  $N = 4$  multiplets includes five  $N = 4$  vector multiplets with coupling constant  $g_{YM}^2 = l$ , and the strong coupling limit of each should give a (2,0) 6-dimensional tensor multiplet [1]. However, the 27 vector fields of the  $N = 8$  theory fit into an irreducible representation of  $E_6$ , and so if  $E_6$  symmetry is to be maintained and if some of the vector fields become self-dual 2-form gauge fields in 6 dimensions, then all of them should. Furthermore, all the fields fit into an irreducible multiplet of the  $N = 8$  supersymmetry algebra, so that if the

32 supersymmetries survive at strong coupling and some of the fields become 6-dimensional, then the whole theory should become 6-dimensional. As in the gauge theory case, all the scalar fields should survive at strong coupling and so should lift to 42 scalars in 6 dimensions. If the  $D = 6$  theory were conformally invariant, then the  $D = 5$  gravitational coupling  $l$  could arise from the radius of the circle  $R$ .

This then suggests that the strong coupling dual field theory should be a superconformal theory in six dimensions with 32 ordinary supersymmetries (and a further 32 conformal supersymmetries), and should have 42 scalars and 27 self-dual 2-form gauge fields. The unique supergravity theory in 6 dimensions with 32 supersymmetries fails on every one of these requirements with the obvious exception of having 32 supersymmetries. Remarkably, such a conformal supermultiplet in six dimensions with 32 supersymmetries,  $Sp(4)$  R-symmetry, 42 scalars and 27 self-dual 2-form gauge fields *does* exist and so is an immediate candidate for a strong coupling dual.

The multiplet has (4,0) supersymmetry in  $D = 6$  and was studied in [1]. Instead of a graviton, it has an exotic fourth-rank tensor gauge field  $C_{MNPQ}$  with the algebraic properties of the Riemann tensor and the gauge symmetry

$$\delta C_{MNPQ} = \partial_{[M}\chi_{N]PQ} + \partial_{[P}\chi_{Q]MN} - 2\partial_{[M}\chi_{NPQ]} \quad (2)$$

with parameter  $\chi_{MPQ} = -\chi_{MQP}$ . (Similar gauge fields were considered in  $D = 4$  in [12].) The invariant field strength is

$$G_{MNPQRS} = \frac{1}{36}(\partial_M\partial_S C_{NPRS} + \dots) = \partial_{[M}C_{NP][QR,S]} \quad (3)$$

and in the (4,0) multiplet it satisfies the self-duality constraint

$$G_{MNPQRS} = \frac{1}{6}\epsilon_{MNP TUV}G^{TUV}{}_{QRS} \quad (4)$$

or  $G = *G$ . In addition, there are 27 2-form gauge-fields with self-dual field strengths, 42 scalars, 48 symplectic Majorana-Weyl fermions and, instead of gravi-

mini, 8 spinor-valued 2-forms  $\Psi_{MN}^\alpha$  which satisfy a symplectic Majorana-Weyl constraint and have self-dual field strengths. The fermionic gauge symmetry is of the form

$$\delta\Psi_{MN}^a = \partial_{[M}\varepsilon_{N]}^a \quad (5)$$

with parameter a spinor-vector  $\varepsilon_N^{\alpha a}$ . The free theory based on this multiplet is a superconformally invariant theory, with conformal supergroup  $OSp^*(8/8)$  [1]. This has bosonic subgroup  $USp(8) \times SO^*(8) = Sp(4) \times SO(6, 2)$  and 64 fermionic generators, consisting of the 32 supersymmetries of the (4,0) superalgebra and 32 conformal supersymmetries.

It is remarkable that in going from  $D = 5$  to  $D = 6$ , the vector gauge fields  $A_\mu$  are lifted to 2-forms  $B_{MN}$ , the gravitini  $\psi_\mu$  are lifted to spinor-valued 2-forms  $\Psi_{MN}$  and the graviton  $h_{\mu\nu}$  is lifted to the gauge field  $C_{MNPQ}$ , with these  $D = 6$  gauge-fields all satisfying self-duality constraints. Electrically charged 0-branes and magnetic strings in  $D = 5$  lift to BPS self-dual strings in  $D = 6$ . In [1] it was shown that the dimensional reduction of the free (4,0) theory on a circle indeed gives the linearised  $D = 5$ ,  $N = 8$  supergravity theory, with gravitational coupling (Planck length) given by the circle radius  $l = R$ . However, there are no covariant local interactions in  $D = 6$  for this multiplet that could give rise to the  $D = 5$  supergravity interactions.

There is then a close analogy between the  $D = 5$ ,  $N = 4$  Yang-Mills theory and the  $D = 5$ ,  $N = 8$  supergravity. The linearised versions of these theories both arise from the dimensional reduction of a superconformal field theory in  $D = 6$ , with the dimensional  $D = 5$  coupling constant arising from the radius of compactification, so that the strong coupling limit of the  $D = 5$  free theories is a decompactification to  $D = 6$ . For the interacting  $D = 5$  Yang-Mills theory, there are a number of arguments to support the conjecture that its strong coupling limit should be an interacting superconformal theory in  $D = 6$  with (2,0) supersymmetry, even though such a theory has not been constructed and indeed cannot have a conventional field theory formulation. This is not without precedent; in  $D = 2$ , many ‘conformal field

theories' do not appear to have any formulation as conventional field theories, and in any dimension, the absence of asymptotic particle states suggests that these are better formulated not in terms of field theory but in terms of correlation functions of operators occurring in various representations of the conformal group. This led to the conjecture of [1] that the situation for  $D = 5$  supergravity is similar to that of  $D = 5$  super-Yang-Mills, and that a certain strong coupling limit of the interacting supergravity theory should give an interacting theory whose free limit is the  $(4,0)$  theory in  $D = 6$ . In this case there is no analogue of the M5-brane argument to support this, although the M5-brane case does set a suggestive precedent.

The limiting theory should have some magical form of interactions which give the non-polynomial supergravity interactions on reduction. It could be that these are some non-local or non-covariant self-interactions of the  $(4,0)$  multiplet, or it could be that other degrees of freedom might be needed; one candidate might be some form of string field theory. However, if a strong coupling limit of the theory does exist that meets the requirements assumed here, then the limit must be a  $(4,0)$  theory in six dimensions, and this would predict the existence of M-interactions arising from the strong coupling limit of the supergravity interactions. Although it has not been possible to prove the existence of such a limit, it is remarkable that there is such a simple candidate theory for the limit with so many properties in common with the  $(2,0)$  limit of the  $D = 5$  gauge theory. Conversely, if there is a 6-dimensional phase of M-theory which has  $(4,0)$  supersymmetry, then its circle reduction to  $D = 5$  must give an  $N = 8$  supersymmetric theory and the scenario described here should apply.

In the context in which the  $D = 5$  supergravity is the massless sector of M-theory on  $T^6$ , then the  $D = 6$  superconformal theory would be a field theory sector of a new 6-dimensional superconformal phase of M-theory. This could tell us a great deal about M-theory: it would be perhaps the most symmetric phase of M-theory so far found, with a huge amount of unbroken gauge symmetry, and would be a phase that is not well-described by a conventional field theory at low energies, so that it could give new insights into the degrees of freedom of M-theory.

The various strong-coupling limits of the 5-dimensional string theory (obtained by compactifying M-theory on  $T^6$ ) that were analysed in [10, 11] in which 0-branes become massless all correspond to limits of the (4,0) theory in which some string tensions become infinite and others approach zero, and an understanding of such tensionless strings will be an important part of understanding the (2,0) and (4,0) theories.

Particularly intriguing are the consequences for gravity. In  $D = 5$ , gravity is described geometrically in terms of a metric  $g_{\mu\nu}$ , but at strong coupling it is described instead in terms of the gauge field  $C_{MNPQ}$  in  $D = 6$  (at least in the free case). This suggests the possibility of some new structure which reduces to Riemannian geometry but which is more general and is the appropriate language for describing gravity beyond the Planck scale. For example, while  $g_{\mu\nu}$  provides a norm for vectors and a notion of length,  $C_{MNPQ}$  could provide a norm  $C_{MNPQ}\omega^{MN}\omega^{PQ}$  for 2-forms  $\omega^{MN}$  and hence a notion of area that is not derived from a concept of length. It may be that the interacting theory is not described in conventional  $D = 6$  spacetime at all, but in some other arena.

There seem to be three main possibilities. The first is that there is no interacting version of the (4,0) theory, that it only exists as a free theory, and that the limit proposed in [1] only exists for the free  $D = 5$  theory. The second is that an interacting form of the theory does exist in 6 spacetime dimensions, with  $D = 6$  diffeomorphism symmetry. The absence of a spacetime metric means that such a generally covariant theory would be of an unusual kind. The third and perhaps the most interesting possibility is that the theory that reduces to the interacting supergravity in  $D = 5$  is not a diffeomorphism-invariant theory in six spacetime dimensions, but is something more exotic. There are some suggestions for this from the linear theory, as will now be discussed.

For gravity in any dimension, the full non-linear gauge symmetry is

$$\delta g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} \tag{6}$$



If the metric is written as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad (7)$$

in terms of a fluctuation  $h_{\mu\nu}$  about some background metric  $\bar{g}_{\mu\nu}$  (e.g. a flat background metric) then two main types of symmetry emerge. The first consists of ‘background reparameterizations’

$$\delta\bar{g}_{\mu\nu} = 2\bar{\nabla}_{(\mu}\xi_{\nu)}, \quad \delta h_{\mu\nu} = \mathcal{L}_\xi h_{\mu\nu} \quad (8)$$

where  $\bar{\nabla}$  is the background covariant derivative with connection constructed from  $\bar{g}_{\mu\nu}$ , while  $h_{\mu\nu}$  transforms as a tensor ( $\mathcal{L}_\xi$  is the Lie derivative with respect to the vector field  $\xi$ ), as do all other covariant fields. The second is the ‘gauge symmetry’ of the form

$$\delta\bar{g}_{\mu\nu} = 0, \quad \delta h_{\mu\nu} = 2\nabla_{(\mu}\zeta_{\nu)} \quad (9)$$

in which  $h_{\mu\nu}$  transforms as a gauge field and the background is invariant. There is in addition the standard shift symmetry under which

$$\delta\bar{g}_{\mu\nu} = \alpha_{\mu\nu}, \quad \delta h_{\mu\nu} = -\alpha_{\mu\nu} \quad (10)$$

In terms of the full metric  $g_{\mu\nu}$ , there is no shift symmetry and a unique gauge symmetry (6); the various types of symmetry (8),(9),(10) are an artifice of the background split. The shift symmetry is a signal of background independence and plays an important role in the interacting theory.

The linearised  $D = 5$  supergravity theory has both background reparameterization and gauge invariances given by the linearised forms of (8),(9) respectively, and both of these have origins in  $D = 6$  symmetries of the free (4,0) theory. The background reparameterization invariance lifts to the linearised  $D = 6$  background

reparameterization invariance

$$\delta\bar{g}_{MN} = 2\partial_{(M}\xi_{N)}, \quad \delta C_{MNPQ} = \mathcal{L}_\xi C_{MNPQ} \quad (11)$$

with the transformations leaving the flat background metric  $\bar{g}_{MN}$  invariant forming the  $D = 6$  Poincaré group. The  $D = 5$  gauge symmetry given by the linearised form of (9) arises from the  $D = 6$  gauge symmetry (2) with  $\delta\bar{g}_{MN} = 0$  and the parameters related by  $\zeta^\mu = \chi^{55\mu}$ . The  $D = 6$  theory has no analogue of the shift symmetry, and the emergence of that symmetry on reduction to  $D = 5$  and dualising to formulate the theory in terms of a graviton  $h_{\mu\nu}$  comes as a surprise from this viewpoint.

The gravitational interactions of the full supergravity theory in  $D = 5$  are best expressed geometrically in terms of the total metric  $g_{\mu\nu}$ . If an interacting form of the (4,0) theory exists that reduces to the  $D = 5$  supergravity, it must be of an unusual kind. One possibility is that there is no background metric of any kind in  $D = 6$ , and the full theory is formulated in terms of a total field corresponding to  $C$ , with a spacetime metric emerging only in a particular background  $C$  field and a particular limit corresponding to the free theory limit in  $D = 5$ .

It is not even clear that the interacting theory should be formulated in a  $D = 6$  spacetime. In  $D = 5$ , the diffeomorphisms act on the coordinates as

$$\delta x^\mu = \xi^\mu \quad (12)$$

In the (4,0) theory, the parameter  $\xi^\mu$  lifts to a parameter  $\chi^{MNP}$ . If the coordinate transformations were to lift, it could be to something like a manifold with coordinates  $X^{MNP}$  transforming through reparameterisations

$$\delta X^{MNP} = \chi^{MNP} \quad (13)$$

with the  $D = 5$  spacetime arising as a submanifold with  $x^\mu = X^{55\mu}$ .

Similar considerations apply to the local supersymmetry transformations. In  $D = 5$ , the local supersymmetry transformations in a supergravity background give rise to ‘background supersymmetry transformations’ with symplectic Majorana spinor parameters  $\epsilon^{\alpha a}$  (where  $\alpha$  is a  $D = 5$  spinor index and  $a = 1, \dots, 8$  labels the 8 supersymmetries) in which the gravitino fluctuation  $\psi_\mu^a$  transforms without a derivative of  $\epsilon^a$ , and ‘gauge supersymmetries’ with spinor parameter  $\varepsilon^{\alpha a}$  under which

$$\delta\psi_\mu^a = \partial_\mu \varepsilon^a + \dots \quad (14)$$

The background symmetries preserving a flat space background form the  $D = 5$  super-Poincaré group. In the free theory, the  $D = 5$  super-Poincaré symmetry lifts to part of a  $D = 6$  super-Poincaré symmetry with  $D = 5$  translation parameters  $\xi^\mu$  lifting to  $D = 6$  ones  $\Xi^M$  and supersymmetry parameters  $\epsilon$  lifting to  $D = 6$  spinor parameters  $\hat{\epsilon}$ . The corresponding  $D = 6$  supersymmetry charges  $Q$  and momenta  $P$  are generators of the  $(4,0)$  super-Poincaré algebra with

$$\{Q_\alpha^a, Q_\beta^b\} = \Omega^{ab}(\Pi_+ \Gamma^M C)_{\alpha\beta} P_M \quad (15)$$

where  $\Pi_\pm$  are the chiral projectors

$$\Pi_\pm = \frac{1}{2}(1 \pm \Gamma^7) \quad (16)$$

$\alpha, \beta$  are  $D = 6$  spinor indices and  $a, b = 1, \dots, 8$  are  $USp(8)$  indices, with  $\Omega^{ab}$  the  $USp(8)$ -invariant anti-symmetric tensor. This is in turn part of the  $D = 6$  superconformal group  $OSp^*(8/8)$ .

The  $D = 5$  gauge symmetries including those with parameters  $\zeta^\mu, \varepsilon^a$  satisfy a local algebra whose global limit is the  $D = 5$  Poincaré algebra, but the  $D = 6$  origin of this (at least in the free theory) is an algebra including the generators  $\mathcal{Q}_{\alpha M}^a$  of the fermionic symmetries with parameter  $\varepsilon_N^\alpha$  and the generators  $\mathcal{P}^{MNP}$  of

the bosonic symmetries with parameter  $\chi^{MNP}$ . The global algebra is of the form

$$\{\mathcal{Q}_{\alpha N}^a, \mathcal{Q}_{\beta P}^b\} = \Omega^{ab}(\Pi_+ \Gamma^M C)_{\alpha\beta} \mathcal{P}_{(NP)M} \quad (17)$$

In the dimensional reduction, the  $D = 5$  superalgebra has charges  $Q_\alpha^a = \mathcal{Q}_{\alpha 5}^a$ ,  $P_\mu = \mathcal{P}_{55\mu}$ .

Supersymmetry provides a further argument against the possibility of a background metric playing any role in an interacting (4,0) theory in  $D = 6$ . The  $D = 5$  supergravity can be formulated in an arbitrary supergravity background, but these cannot be lifted to  $D = 6$  (4,0) backgrounds involving a background metric as there is no (4,0) multiplet including a metric or graviton. The absence of a (4,0) supergravity multiplet appears to rule out the possibility of a background metric and the standard supersymmetry (15) playing any role in the  $D = 6$  theory. Indeed, the interacting theory (if it exists) should perhaps be a theory based on something like the algebra (17) rather than the super-Poincaré algebra (15).

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